Section 2-1,2, Mathematics 104

Language

If we write a string of symbols

$$4+3\times 2\div 6$$

We need to know the **order of operation**.

The most common convention is that multiplication and division have precedence over addition and subtraction.

$$4+3\times 2\div 6=4+\left(\left(3\times 2\right)\div 6\right)$$

Note that operations of equal precedence are evaluated left to right.

There is not always complete agreement on how to do this. For example, what is?

$$-3^2$$

The question is which has higher precedence, negating a number or exponentiation. Our book considers exponentiation higher so,

$$-3^2=-\left(3^2\right)=-9$$

However some texts might instead consider negation

$$-3^2=\left(-3\right)^2=9$$

You may also run into various programming languages that do things differently.

When in doubt, it is a good idea to remove any ambiguity using parenthesis and brackets:

Example:

$$\left[\left(3+2\right)\times\left(2+5\right)\right]\div 6$$

Occasionally you will see curly braces used, however these usually have a more specific purpose.

$$\{\ \}$$

Inuits have at least 50 words for snow. Similarly because multiplication is so important in mathematics, we have more than one way to express it.

3×4

$3 \cdot 4$

$3(4)$

$(3)(4)$

And when we use a letter to express a number we can leave off any operator at all, eg.

$3x$

$3A$

3π

This brings us to symbols for numbers.

The most often used symbol is x.

When we use the letters x, y, and z we usually mean a **variable**. By variable we mean it can be replaced by many values, just one or even none in some cases. For example:

$x+3 = x+4$

There are no number that we can substitute for x that make this equation true.

We also use letters such as a, b and c to mean a number but not one that can vary. We call these **constants**.

A number or constant multiplied by a variable is called a **coefficient.**
Example:

$ax^2 + 5x + c$ x is a variable a, and c are constants but a and 5 are coefficients.

Sometimes mathematics can be confusion because the same symbols can mean different things in different contexts.

Example:

(5,6)

can be an order pair representing a point on an *X Y* coordinate system.

It could also mean an **open** line segment between the points 5 and 6 on a number line. Open here means that the end points are not included.

In an arithmetic expression, the symbols connected by addition or subtraction are called **terms.**

Example:

$x + 2y + 4$ - x, $2y$ and 4 are each a term.

Sometimes we use letters that are meant to be shorthand for what they stand for, for example:

$d = vt$

Another example from physics, Newton's law of gravity looks like this:

$$F = \frac{GmM}{r^2}$$

In this expression F is the force of gravity between to objects.
M and m are the masses of the two objects
r is the distance between them
G is the universal gravitational constant

F, M, m and r are variables.

When you have an equation of the form

Variable = Expression with variables
Example:
$y = 5x + 3$

We call x the **independent** variable, since you are free to choose its value and
y is the **dependent** variable, since once you choose x, then y is determined.

Visualizing Math

Different people will learn to understand or remember mathematics in different ways.

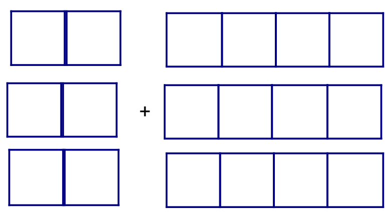
While it is know that people will use different **modalities,** studies have surprisingly shown no difference is learning when different modalities are used in a classroom.

https://www.edweek.org/tm/articles/2010/02/17/tln_wolpertgawron_learningstyles.html

My guess is that students in a mathematics class room take whatever is given to them and as a part of the learning process convert the lesson into a modality that works best for them.

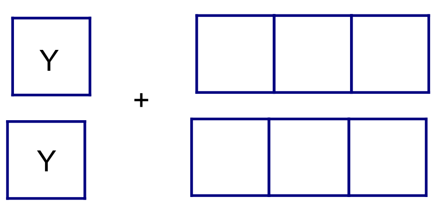
Here's an example of the distributive law shown visually

$3(2+4)=3\cdot 2+3\cdot 4$



When we use variables, the same Idea can be useful.

$2(y+3)$



Manipulating Mathematical Expressions

Once we understand what a mathematical expression means, we need to know how to validly manipulate it. One guide is the properties of numbers.

1. The associate properties $(a+b)+c=a+(b+c)$ $(ab)c=a(bc)$

2. The commutative properties $a+b=b+a$ $ab=ba$

3. The identity properties $a+0=0+a=a$ $a\cdot 1=1\cdot a=a$

4. The inverse properties $a+{}^{-}a=0$ $a\cdot\frac{1}{a}=1$

5. The distributive property $a(b+c)=ab+ac$

Using these properties we can manipulate and simplify expressions.

Have students do these:

Example:

$4(3+x)=4\cdot 3+4\cdot x=12+4x$ the distributive property

$5+2x+7+x^2-3x=x^2+2x-3x+5=x^2-x+5=$ commutative and associate properties

$-3(-5x)$

$\frac{5x}{3}\cdot\frac{3}{5}$

$5xy+1-xy$

$12-x^2+3x-5$

$7x-3-2x+5$

Exponents in Algebraic expressions

Examples:

$x^4 = x \cdot x \cdot x \cdot x$

$(x+1)^2 = (x+1) \cdot (x+1)$

$(-3x)^3 = (-3x)(-3x)(-3x) = ?$

Evaluating Algebraic Expressions, by plugging in values for the variables

Examples:

If we have an expression:

$x^2 + 5$

We can "Evaluate with x=-3" as follows:

$(-3)^2 + 5 = -3 \cdot -3 + 5 = 9 + 5 = 14$

Example:

$y - 2(x+5)$

Let x=-3 and y=5

$5 - 2(-3+5) = 5 - 2(2) = 5 - 4 = 1$

Example:
$y^2 - 3y$

Let y=5

$5^2 - 3 \cdot 5 = 25 - 15 = 10$

Note: we can also use the distributive property here:

$y^2 - 3y = y(y-3) = 5(5-3) = 5 \cdot 2 = 10$

Example:

$|y-x|$

Let x=4 and let y=-6

$|-6-4|=|-10|=10$

Example:

x= -5, y= -2, z=3

$\frac{y+2z}{5y-xz}$

A Real World Example

$F^\circ=\frac{9}{5}C^\circ+32^\circ$

This formula converts from Celsius to Fahrenheit temperature.

Find the values of F°

when

$C^\circ=0^\circ$
$C^\circ=100^\circ$
$C^\circ=-40^\circ$